

Math 115

Summer 2017

Lecture 6



Cont. with graphing

New Notations & Formulas

\overline{AB} \Rightarrow line segment connecting A to B.

\overleftrightarrow{AB} \Rightarrow line that contains points A and B.

$m \Rightarrow$ slope $m = \frac{y_1 - y_2}{x_1 - x_2}$, $m = \frac{y_2 - y_1}{x_2 - x_1}$

$M \Rightarrow$ Midpoint $M \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$

$d \Rightarrow$ distance $d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$

Ex: $A(-2, 3)$, $B(0, 7)$

1) Draw \overline{AB} ← line segment

2) Find slope

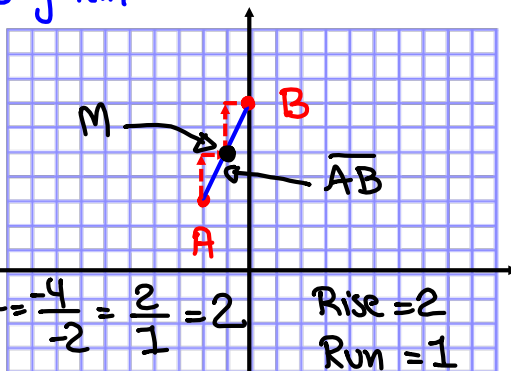
$$m = \frac{y_1 - y_2}{x_1 - x_2} = \frac{3 - 7}{-2 - 0} = \frac{-4}{-2} = \frac{2}{1} = 2$$

Rise = 2
Run = 1

3) Find the midpoint

$$M\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = M\left(\frac{-2 + 0}{2}, \frac{3 + 7}{2}\right) \Rightarrow M(-1, 5)$$

4) Find the distance $d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2} = \sqrt{(-2 - 0)^2 + (3 - 7)^2}$
 $= \sqrt{4 + 16} = \sqrt{20} \approx \boxed{4.4}$



$A(6, 0)$, $B(0, 8)$

1) Draw \overline{AB}

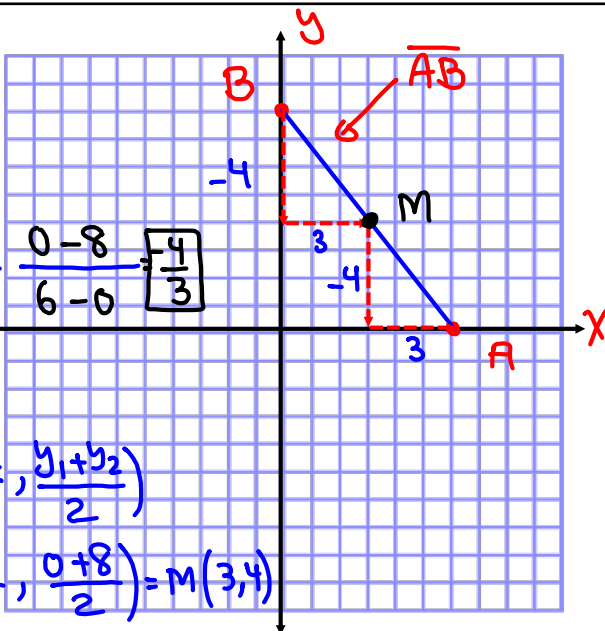
2) Slope $m = \frac{y_1 - y_2}{x_1 - x_2} = \frac{0 - 8}{6 - 0} = \boxed{-\frac{4}{3}}$

Rise = -4, Run = 3

3) Midpoint $M\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

$$M\left(\frac{6 + 0}{2}, \frac{0 + 8}{2}\right) = M(3, 4)$$

4) Distance $d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$
 $= \sqrt{(6 - 0)^2 + (0 - 8)^2} = \sqrt{100} = \boxed{10}$



$A(-2, 5)$, $B(2, -5)$

① Draw \overline{AB}

2) find slope m

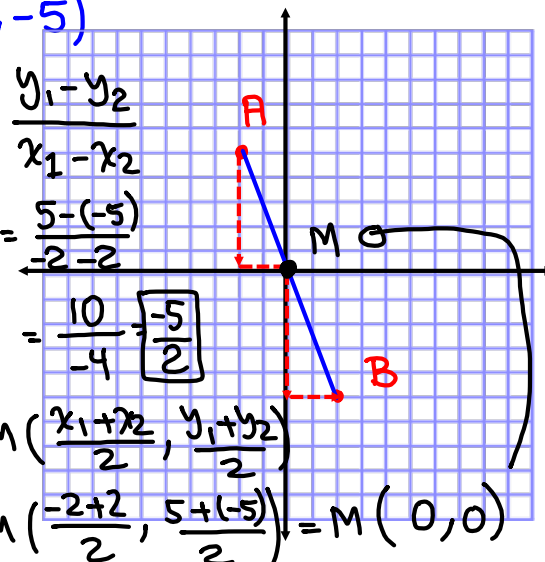
$$m = \frac{y_1 - y_2}{x_1 - x_2} = \frac{5 - (-5)}{-2 - 2} = \frac{10}{-4} = \boxed{-\frac{5}{2}}$$

3) find Midpoint M

$$M\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = M\left(\frac{-2 + 2}{2}, \frac{5 + (-5)}{2}\right) = M(0, 0)$$

4) find distance d

$$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2} = \sqrt{(-2 - 2)^2 + (5 - (-5))^2} = \sqrt{(-4)^2 + (10)^2} = \sqrt{116} \approx \boxed{10.8}$$



$A(-4, -3)$, $B(2, 4)$

① Draw \overline{AB}

② Slope m

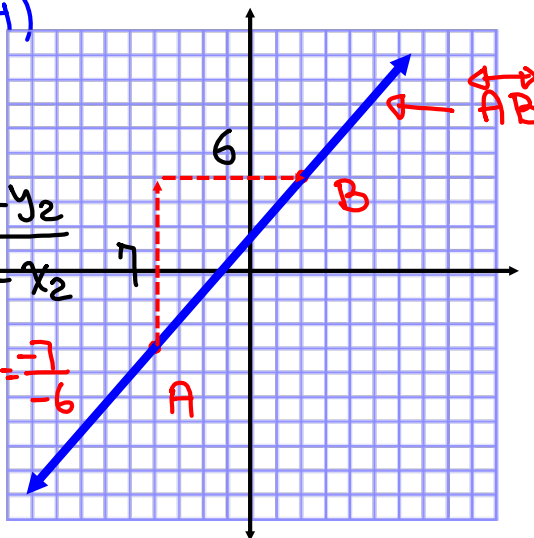
$$m = \frac{y_1 - y_2}{x_1 - x_2}$$

$$m = \frac{-3 - 4}{-4 - 2} = \frac{-7}{-6} = \boxed{\frac{7}{6}}$$

Rise 7

Run 6

$$= \boxed{\frac{7}{6}}$$



$m > 0 \Leftrightarrow$ Increasing line

$m < 0 \Leftrightarrow$ Decreasing line

$m = 0 \Leftrightarrow$ Horizontal line

m is undefined \Leftrightarrow vertical line

Graph $x=5$, $y=-4$ in the Same Coordinate System.

$$x=5$$

x -only

V.L.

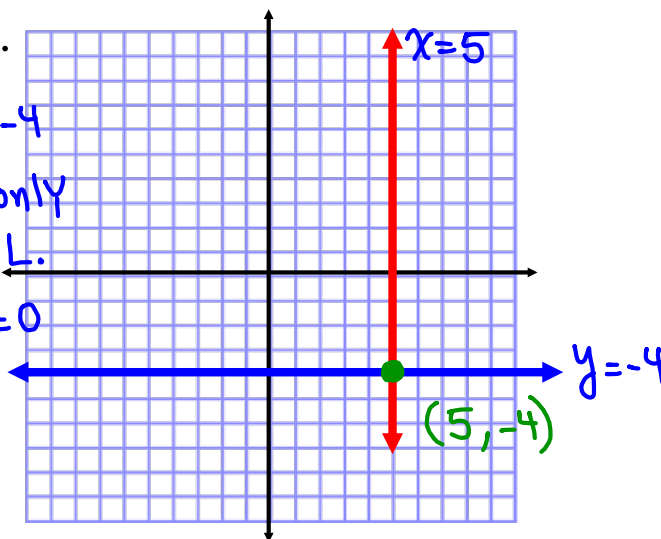
No Slope

$$y=-4$$

y -only

H.L.

$$m=0$$



Graph $x+3=0$, $y-6=0$ in the Same Coordinate System.

$$x+3=0$$

$$x=-3$$

x -only

V.L.

No slope

Undefined Slope

$$y-6=0$$

$$y=6$$

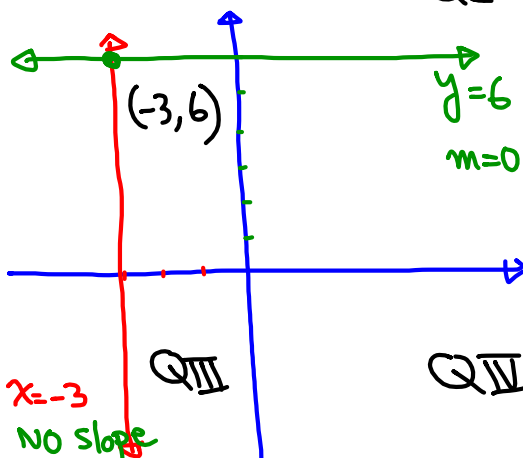
y -only

H.L.

$$m=0$$

QII

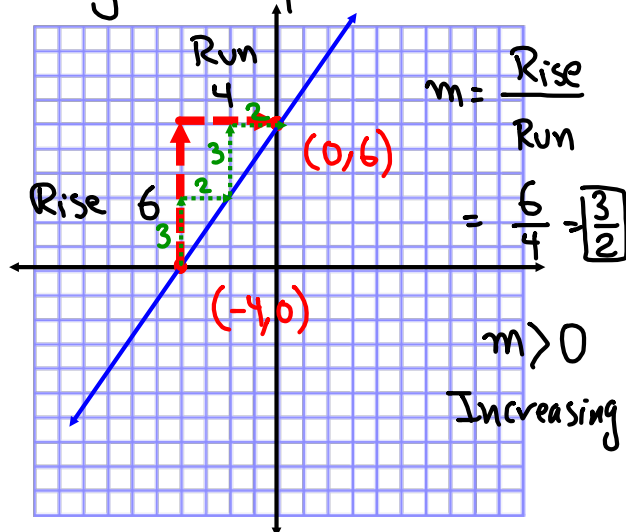
QI



Graph $3x - 2y = -12$ by intercept method.

x	y
0	6
-4	0

Intercept Method



Graph $3x + 5y = -30$ by intercept method.

use the graph to find its slope.

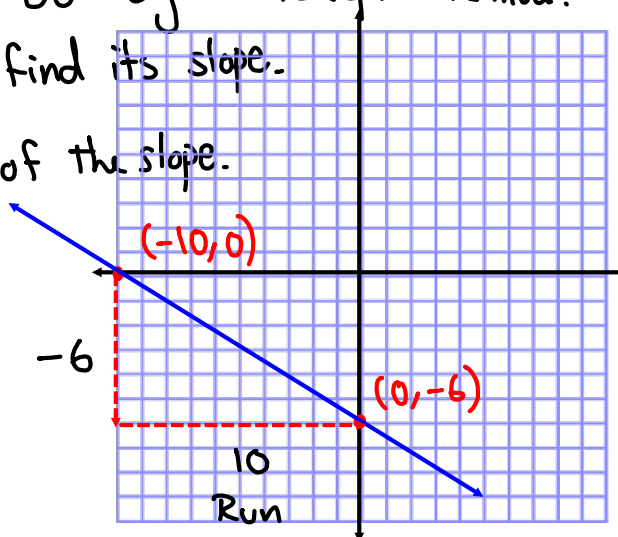
Show rise & run of the slope.

x	y
0	-6
-10	0

Rise -6

$$m = \frac{-6}{10} = \boxed{\frac{-3}{5}}$$

$m < 0$ Decreasing



Graphing Slope-Int. Form eqn of a line

$$y = mx + b$$

Y-Int $(0, b)$, slope m

1) Plot Y-Int

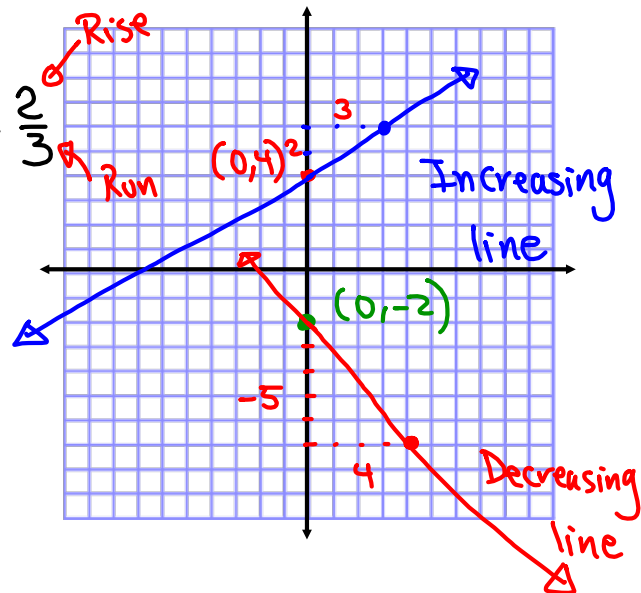
2) From the Y-Int, use rise & run of the slope to get a second point.

3) Connect the points & extend.

$$y = \frac{2}{3}x + 4$$

Y-Int $(0, 4)$

$$m = \frac{2}{3}$$



$$y = -\frac{5}{4}x - 2$$

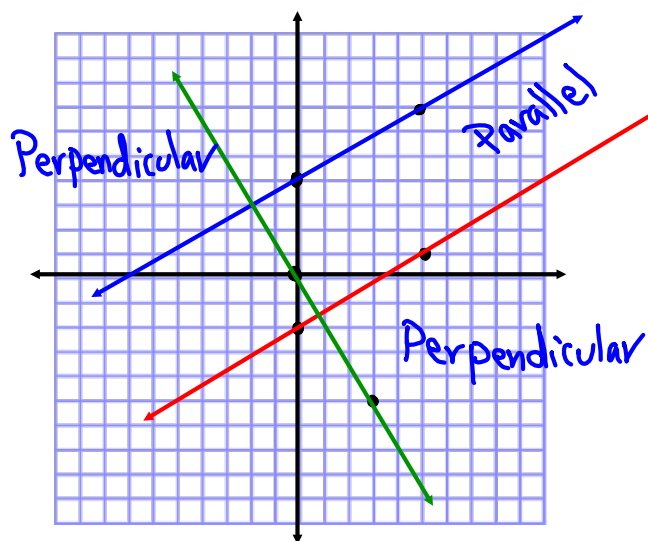
Y-Int $(0, -2)$

$$m = -\frac{5}{4}$$

Rise

Run

Graph $y = \frac{3}{5}x + 4$, $y = \frac{3}{5}x - 2$, $y = -\frac{5}{3}x$



Simplify:

$$\frac{-|-5|(3^2 - 4^2)}{\sqrt{100} - 2^2 - 1}$$

$$= \frac{-5(9 - 16)}{10 - 4 - 1} = \frac{-5(-7)}{6 - 1}$$

$$= \frac{35}{5} = \boxed{7}$$

Evaluate

$$\frac{-b - \sqrt{b^2 - 4ac}}{2a}$$

for $a=1$, $b=8$, $c=16$

$$= \frac{-8 - \sqrt{8^2 - 4(1)(16)}}{2(1)}$$

$$= \frac{-8 - \sqrt{64 - 64}}{2}$$

$$= \frac{-8}{2} = \boxed{-4}$$

Simplify

$$-\frac{3}{5}(10x^2 - 15x + \frac{5}{3}) + 6x^2 - 9x$$

$$= \cancel{\frac{-3}{5}} \cdot \cancel{10}^2 x^2 - \cancel{\frac{3}{5}} \cdot \cancel{(-15)}^3 x - \frac{3}{5} \cdot \frac{5}{3} + 6x^2 - 9x$$

$$= -6x^2 + 9x - 1 + 6x^2 - 9x$$

$$= \boxed{-1}$$

$$3x^2 = 10 - x$$

Translate only:

3 times square
of some number

is equal to

the number less
than 10.

① 3.2% of what number is 160?

$$\frac{3.2}{100} = \frac{160}{x}$$

$$3.2x = 100(160)$$

$$\boxed{x = 5000}$$

3.2% of 5000
is 160.

② 3 cups of Sugar for 10 muffins.

How many cups of sugar for 45 muffins?

$$\frac{3 \text{ cups Sugar}}{10 \text{ Muf.}} = \frac{x \text{ cups Sugar}}{45 \text{ muf.}}$$

$$10x = 3(45)$$

$$x = 13.5$$

13.5 cups of Sugar

Solve & identify the type of eqn:

$$\frac{2}{3}(x-4) = \frac{1}{2}(x+3) + 1$$

$$\text{LCD} = 6$$

$$\cancel{6} \cdot \frac{2}{\cancel{3}}(x-4) = \cancel{6} \cdot \frac{1}{\cancel{2}}(x+3) + 6 \cdot 1$$

$$4(x-4) = 3(x+3) + 6$$

$$4x - 16 = 3x + 9 + 6$$

$$4x - 3x = 15 + 16$$

$$\boxed{x=31}$$

$$\{31\}$$

Conditional

Solve

$$.05x + .1(2x + 7) = 3.7$$

$$.05x + .2x + .7 = 3.7$$

$$.25x = 3.7 - .7$$

$$.25x = 3$$

$$x = \frac{3}{.25}$$

$$\boxed{x=12}$$

$$\{12\}$$

Conditional eqn.

Solve

$$2(x-7) + 4 \leq 5x + 11$$

$$2x - 14 + 4 \leq 5x + 11$$

$$2x - 5x \leq 11 + 10$$

$$-3x \leq 21$$

$$\textcircled{1} \text{ S.B.N. } \{x \mid x \geq -7\}$$

$$x \geq -7$$

$$\textcircled{2} \text{ Graph}$$

$$\textcircled{3} \text{ I.N. } [-7, \infty)$$



Solve

$$-4 < 5x + 1 \leq 41$$

$$-4 - 1 < 5x + 1 - 1 \leq 41 - 1$$

$$-5 < 5x \leq 40$$

$$\textcircled{1} \text{ S.B.N. } \{x \mid -1 < x \leq 8\}$$

$$-\frac{5}{5} < \frac{5}{5}x \leq \frac{40}{5}$$

$$\textcircled{2} \text{ Graph}$$

$$-1 < x \leq 8$$



$$\textcircled{3} \text{ I.N. } (-1, 8]$$

The difference of 5 and twice
Some number exceeds 15.

find all such numbers.

$$5 - 2x > 15$$

$$-2x > 15 - 5$$

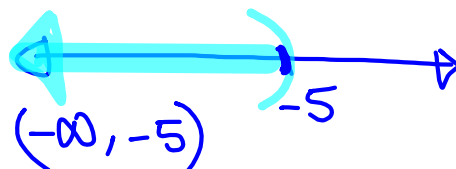
$$-2x > 10$$

$$\frac{-2}{-2}x < \frac{10}{-2}$$

$$x < -5$$

numbers below -5

all numbers
less than -5



$\frac{1}{2}$ of Some number is at most 8 less than twice the number.

Solve & graph.

$$\frac{-3}{-3}x \geq \frac{-16}{-3}$$

$$x \geq \frac{16}{3}$$

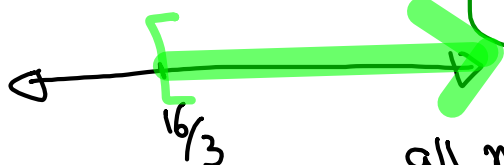
$$\frac{1}{2}x \leq 2x - 8$$

$$\text{LCD} = 2$$

$$x \leq 4x - 16$$

$$x - 4x \leq -16$$

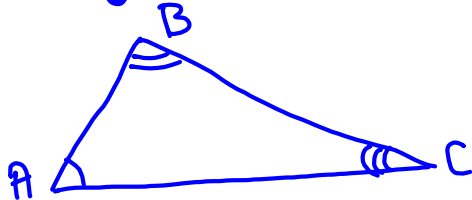
$$-3x \leq -16$$



all numbers that are at least $\frac{16}{3}$.

at most \leq fewer than $<$ at least \geq is less than $<$

Angles & Triangles



$$A + B + C = 180^\circ$$

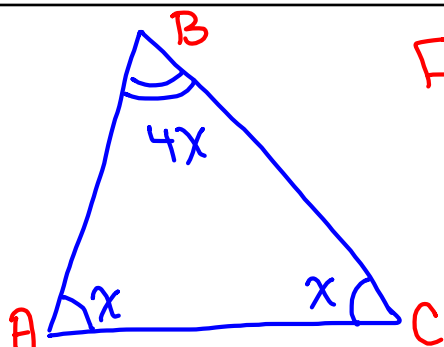
Two angles are Complementary if their total is 90° .

Two angles are Supplementary if their total is 180° .

Angle	Complement	Supplement
x	$90 - x$	$180 - x$
10°	80°	170°
55°	35°	125°

Two angles in triangle ABC are equal

The third angle is 4 times the measure of one of equal angles. Find all three angles.



Fact

$$A + B + C = 180$$

$$x + 4x + x = 180$$

$$6x = 180$$

$$x = 30$$

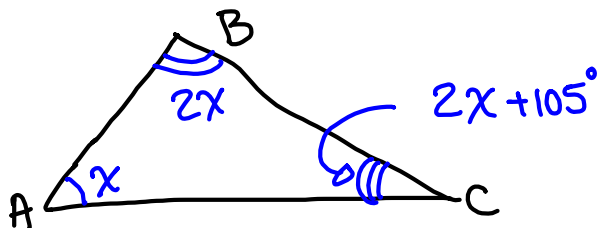
$30^\circ, 30^\circ, \text{ and } 120^\circ$

In triangle ABC,

Angle B is twice angle A.

Angle C is 105° more than angle B.

Find all three angles.



$$A + B + C = 180^\circ$$

$$x + 2x + 2x + 105 = 180$$

$$5x = 75$$

$$x = 15$$

$15^\circ, 30^\circ, \text{ and } 135^\circ$

Two angles are Complementary.
 one of them is 4 times the other one.
 find both angles.

Angle x
 Second Angle $4x$

$$x + 4x = 90$$

$$5x = 90 \quad x = 18$$

$18^\circ \text{ \& } 72^\circ$

Angle x
 Complement $90 - x$

$$x = 4(90 - x)$$

$$x = 360 - 4x$$

$$x + 4x = 360$$

$$5x = 360$$

$$x = 72$$

Two angles are Complementary.

The difference between twice of one of them and 5 times its complement is 40° .
 find both angles.

Angle	Complement	Supplement
x	$90 - x$	$180 - x$

$$2x - 5(90 - x) = 40$$

$$2x - 450 + 5x = 40$$

$$7x = 490$$

$$x = 70$$

$10^\circ \text{ \& } 20^\circ$

Find two complementary angles such that the sum of three times one of them and 7 times its complement is equal to 490° .

Angle	Comp.	Suppl.
x	$90-x$	$180-x$

$$3 \cdot x + 7 \cdot (90 - x) = 490$$

$$x = 35$$

35° & 55°

$$3(90-x) + 7x = 490$$

$$x = 55$$

55° & 35°

Two angles are supplementary. one of them is 20° more than the other one.

find both angles.

Angle	Comp.	Suppl.
x	$90-x$	$180-x$

$$x = 180 - x + 20$$

$$x + x = 200$$

$$2x = 200 \quad x = 100$$

$$180 - x = x + 20$$

$$x = 80$$

100° & 80°

80° & 100°

Find two Supplementary angles such that 6 times one of them is equal to the difference between 1120° and 7 times the Supplement.

Angle	Comp.	Suppl.
x	$90-x$	$180-x$

$$6x = 1120 - 7(180 - x)$$

...

$$x = 140$$

$$140^\circ \text{ \& } 40^\circ$$

Find an angle such that the sum of 3 times its Complement and 2 times its Supplement is 505° .

Angle	Comp.	Suppl.
x	$90-x$	$180-x$

$$3(90-x) + 2(180-x) = 505$$

$$x = 25$$

$$25^\circ$$